ASSIGNMENT SET – I

Mathematics: Semester-III

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-303(Unit-I)

Paper: Stochastic Process and Regression

Answer all the questions

1. Define ‘Transient’ and ‘ergodic’ state.
2. What do you mean by extinction probability?
3. What is the usefulness of Markov chain?

4. Define Markov Chain with example. What do you mean by state and transition probability?

5. What do you mean by transition matrix? State Gambler’s ruin problem and write transition matrix for it.

6. Define Markov Chain. What is the usefulness of Markov chain?

7. State and prove the First Entrance theorem.
8. State and prove Chapman Kolmogorov equation
9. Consider the Markov Chain with transition probability matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Test whether the states are periodic and persistent.

10. State basic postulates of poisson process.

11. Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit leaving the *n*th stage of system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage there is a constant probability q that the digits which enters will be transmitted unchanged (i.e. the digit will remain unchanged when it leaves), and probability p otherwise (i.e. the digit changes when it leaves), $p + q = 1$. Find the one-step transition matrix P, and *n*-step transition matrix P^n . Also find P^n when $n \to \infty$.

12. Show that a 'I'∈ S of a Markov chain is recurrent if and only if $\sum_{n=0}^{\infty} P_{ii}^{(n)} = \infty$.

13. State basic postulates of poisson process.

14. Define the graph of a Markov chain.

15. Considering appropriate assumptions derive the probability generating function for the birth and death process when birth and death rate respectively $n\lambda$ and $n\mu$, n being the population size at time t and λ and μ are the constants . Assume that initial population size is $\boldsymbol{i}$.

16. Show that a I∈ S of a Markov chain is recurrent if and only if $\sum_{n=0}^{\infty} P_{ii}^{(n)} = \infty$.

Find the differential equation for birth and death process and hence derive the generating function.

17. Let $\{X_n : n \geq 0\}$ be a two state Markov chain with state space $S = \{0, 1\}$ and transition matrix $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$. Assuming $X_0 = 0$, show that the expected return time to 0 is 2.5.

18. For a Markov chain with finite state space, the number of stationary distributions can be infinite.

19. Consider a Markov chain with transition probability matrix P is given by $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ for any two states i and j. Let $P_{ij}^{(n)}$ denote the n-step transition probability of going from i and j. Then prove that $P_{11}^{(n)} = \frac{2}{9}$.

20. Define the terms: (i) accessible state, (ii) return state, (iii) periodic state, (iv) aperiodic state.

21. What do you mean by transition matrix?

22. State and prove Chapman-Kolmogorov equation for a homogeneous Markov chain $\{X_n\}$. Suppose a two state homogeneous Markov chain has the following transition probability matrix:

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \qquad 0 \le a, b \le 1, \qquad |1-a-b| < 1.$$

23. Prove that (by using Chapman-Kolmogorov equation) the n-step transition probability matrix P(n) is given

$$P(n) = \begin{bmatrix} \frac{b+a(1-a-b)^n}{a+b} & \frac{a-a(1-a-b)^n}{a+b} \\ \frac{b-b(1-a-b)^n}{a+b} & \frac{a+b(1-a-b)^n}{a+b} \end{bmatrix}.$$

24. Deduce the forward diffusion equation for the Wiener process. Also, write the backward diffusion equation from the deduced equation.

25. State birth and death process. Find the differential-difference equation for birth and death process.

26. Let $\{X_n, n \ge 0\}$ be a branching process. Show that if $m = E(X_1) = \sum_{k=0}^{\infty} k\, p_k$ and $\sigma^2 = Var(X_1)$ then $E(X_n) = m^n$ and

$$Var(x_n) = \begin{cases} \frac{m^{n-1}(m^n-1)}{m-1} \sigma^2, & \text{if } m \ne 1 \\ n\,\sigma^2, & \text{if } m = 1 \end{cases}$$

27. Deduce the forward diffusion equation for the Wiener process. Also, write the backward diffusion equation from the deduced equation.

28. Prove that the state j is persistent if and only if

$$\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty.$$

29. State birth and death process. Find the differential-difference equation for birth and death process.

30. State and prove Chapman-Kolmogorov equation for a homogeneous Markov chain $\{X_n\}$. Suppose a two state homogeneous Markov chain has the following transition probability matrix:

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0 \le a, b \le 1, \quad |1-a-b| < 1.$$

Prove that (by using Chapman-Kolmogorov equation) then-step transition probability matrix P(n) is given

$$P(n) = \begin{bmatrix} \frac{b+a(1-a-b)^n}{a+b} & \frac{a-a(1-a-b)^n}{a+b} \\ \frac{b-b(1-a-b)^n}{a+b} & \frac{a+b(1-a-b)^n}{a+b} \end{bmatrix}$$

31. Let $\{X_n, n \ge 0\}$ be a branching process. Show that if $m = E(X_1) = \sum_{k=0}^{\infty} k\, p_k$ and $\sigma^2 = Var(X_1)$ then $E(X_n) = m^n$ and

$$Var(x_n) = \begin{cases} \frac{m^{n-1}(m^n - 1)}{m-1} & \sigma^2, \text{if } m \ne 1 \\ n\,\sigma^2, & \text{if } m = 1 \end{cases}$$

32. Starting from the probability-generating function of the birth and death process find the probability of ultimate extinction in the case of the linear growth process starting with *i* individuals at time 0.

33. Consider a communication system which transmits the two digits 0 and 1 through several stages. Let$\{X_n, n \ge 1\}$ be the digit leaving the *n*th stage of the system and X_0 be the digit entering the first stage (leaving the 0th stage). At each stage, there is a constant probability q that the digits which enter will be transmitted unchanged (i.e. the digit will remain unchanged when it leaves), and probability *p* otherwise (i.e. the digit changes when it leaves), $p + q = 1$. Find the one-step transition matrix *P*, and n-step transition matrix P^n. Also, find P^n when $n \to \infty$.
34. Let $\{X_n,\ n \ge 1\}$ be a Markov chain having state space $S = \{1,2,3,4\}$ and transition matrix

$$P=\begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

Identify the states as transient, persistent, or ergodic.

35. Define multiple correlation coefficient and partial correlation coefficient.
36. Define stochastic process with example. Classify it with respect to state space and time.
37. Define multiple correlation coefficient, and indicate how it differs from simple correlation coefficients.
38. What is multiple correlation in regression?
39. Let (x_i, y_i, z_i), (i=1, 2, 3, …, n)be a sample of size **n**drawn a population. Find the regression equation of **z** on **x** and **y**.
40. Define multiple and partial co-relation coefficients. Find the relation between them.

41. Define stochastic process with example. Classify it with respect to state space and time.
42. The following constants are obtained from measurements on length in mm (x_1), volume in c.c. (x_2) and weight in gm. (x_3) of 300 eggs.

$$x_1 = 55.95, s_1 = 2.26, r_{12} = 0.578$$
$$x_2 = 51.48, s_2 = 4.39, r_{13} = 0.581$$
$$x_3 = 56.03, s_3 = 4.41, r_{23} = 0.974$$

Obtain the linear regression equation of egg-weight on egg-length and egg-volume. Hence estimate the weight of an egg whose length is 58.0 mm and volume is 52.5 cc.
43. Suppose $\left(x_{1\alpha}, x_{2\alpha}, \dots, x_{p\alpha}\right), \alpha = 1, 2, \dots, n$, is a multivariate sample of size *n*. If X_1 is the value of x_1 obtained from the regression curve of x_1 on $x_2, x_3, \dots, x_3$, then show that $Var(X_1) = Cov(x_1, X_1)$ and

$Var(X_1) = \left(1 - \frac{|R|}{R_{11}}\right) s_1^2$ Where the symbols have their usual meanings.
44. Prove that $1 - r_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$. The symbols have their usual meanings.

45. Show that the generating function $P_n(s)$ for the branching process satisfies the following relations:

$P_n(s) = P_{n-1}(P(s))$
$P_n(s) = P(P_{n-1}(s))$, where $P_1(s) = P(s)$.

46. Define multiple correlation coefficient, and indicate how it differs from simple correlation coefficients.
47. Define stochastic process with example. Classify it with respect to state space and time.
48. Deduce multiple regression equation of x_i on $x_2, x_3, \dots, x_p$ in terms of the means, the standard deviations and the inter-correlations of the variables.
49. Show that the generating function $P_n(s)$ for the branching process satisfies the following relations: (i) $P_n(s) = P_{n-1}(P(s))$ and (ii) $P_n(s) = P(P_{n-1}(s))$, where $P_1(s) = P(s)$.
50. Obtain the multiple regression equation of x_1 on $x_2, x_3, \dots, x_p$ in terms of the means, the standard deviations and the inter correlations of the variables.
51. Prove that

$r_{1.23\dots p} = \left(1 - \frac{|R|}{R_{11}}\right)^{1/2}$ where the symbols have their usual meanings.

52. Obtain the multiple regression equation of x_1 on $x_2, x_3, \dots, x_p$ in terms of the means, the standard deviations and the inter correlations of the variables.

PAPER - MTM-303(Unit-II)

Paper: Cryptography

1. Define the terminology: cryptanalysis and cryptology.
2. What is cryptography and network security?
3. Write the short note on CIA tried.
4. Write the full from of the following.
   DES, AES, RSA, DSA
5. What is mono alphabetic cipher?
6. Using the Hill decryption algorithm, find out the original massage of the cipher 'FKMFIO' using the key $\begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}$
7. Write the algorithm of play fair cipher with an example.
8. Write shot note on Rail Fence cipher. Decrypt the message 'RHAVTNUSREDEAIERIKATSOQR' using a row and column transposition cipher with keyword 453621.
9. What is the polyalphabetic cipher? Taking an example smoothly describe an polyalphabetic algorithm.
10. Design the computer security model. Write down the difference between symmetric and asymmetric cryptography.
11. Describe the Euclidean Algorithm with flowchart taking an example. Given to integers 60,72, Find the integers a and b such that 72a+60b=gcd(72,60)

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